Black Holes

As Information Scramblers

Bhavay Tyagi
University of Houston

Dualities in Physics

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$$\downarrow$$

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Dualities in Physics

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- The existence of common "dynamics" in seemingly different systems hints at the fact that there exists some underlying symmetry which connects the two.
- Most Well Known: Wave-Particle Duality
- We're interested in is the Quantum-Gravity Duality

Dualities in Physics

Quantum ⟷ Gravity

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Quantum ⟺ Gravity

$\sim \hbar \times$ something $\rightarrow$ small

$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \rightarrow$ large

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Probabilistic

Spacetime Curvature

If this is the case, then do we think of spacetime as fluctuating and treat it as probabilistic?

A Brief History*

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The Information Problem

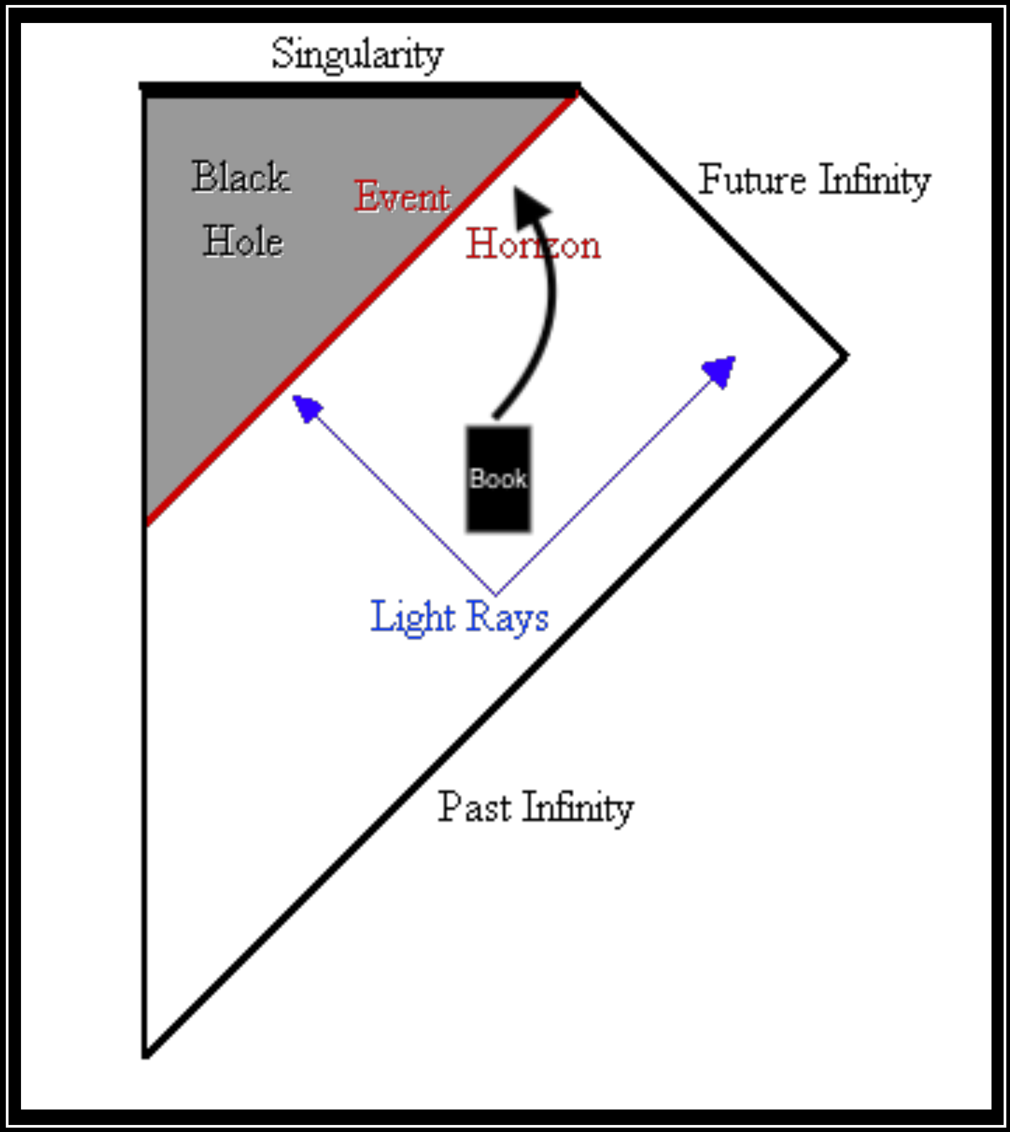
- Imagine a Book written in the Rotokas alphabet. Let's Calculate the Entropy of this Book.

- Possible configurations to arrange 12 letters with an average word length of 5 letters $= {}^{12}C_5$

- Let the Book have $O(3000)$ words. Then $\Omega = ({}^{12}C_5)^{3000}$ possible configurations.

- $S = k_B ln\Omega$ is strictly a positive number.

- Now Imagine dropping this book in a Black Hole.



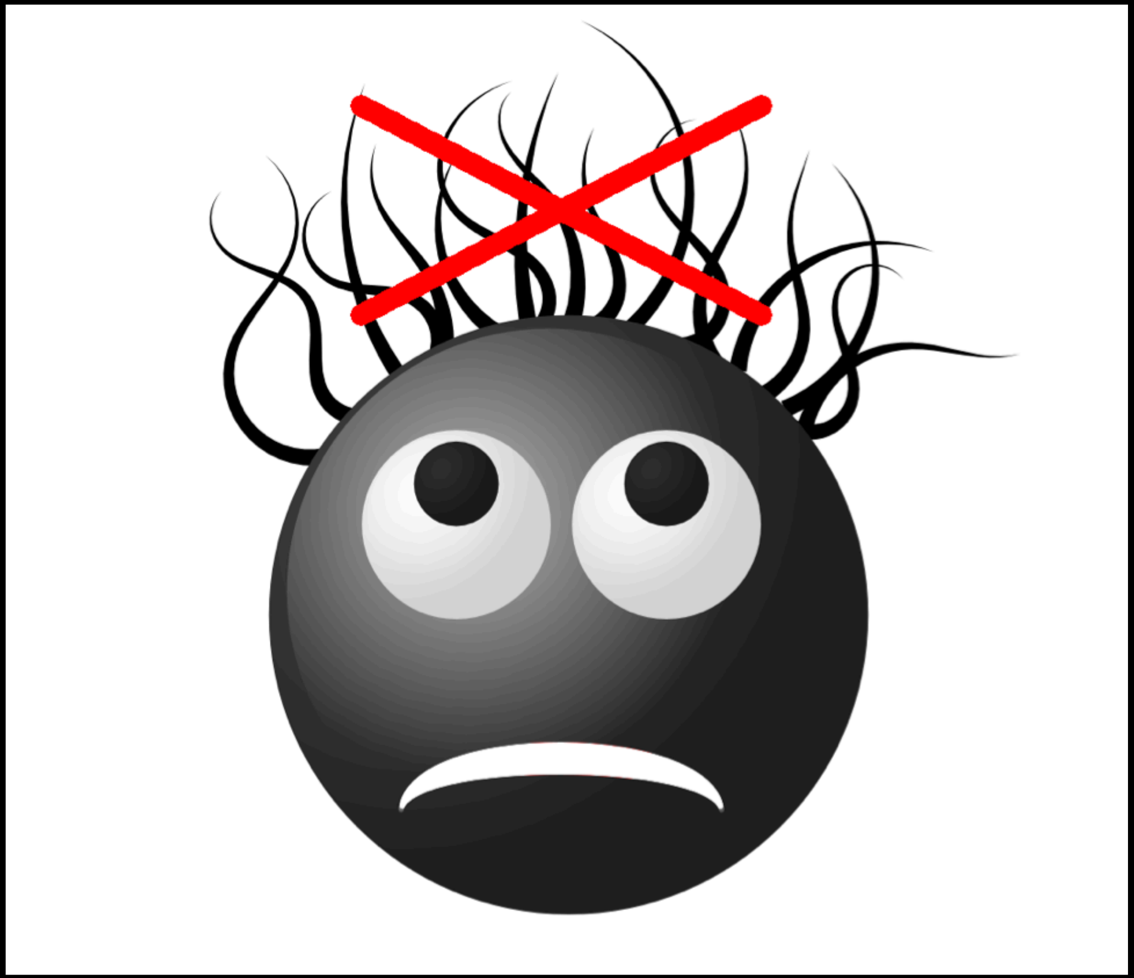
The Information Problem

- $\{M, J, Q\}$ fixes the only possible configuration. Which means $\Omega = 1$

- $\implies S = 0$

- Violation of the 2nd Law $S \geq 0$.

No Hair Theorem



Best test objects?

Why Even Choose Black Holes?

- *Black Holes are astrophysical objects that radiate quanta whose de-Broglie wavelength is comparable to their own size.*

- This property is very "quantum"
  $\lambda_{\text{atom}} \sim 10^{-10} m =$ its size

- Consider a Black Hole of one solar mass M_0. It emits 10^5 photons per second.
  $\lambda_{\text{Hawking}} \sim 1 km =$ its size

- So when viewed from sufficiently far away from the outside they can be treated as quantum objects*.

But?

- Such a Black Hole would take 10^{65} years to evaporate.
- A quantum computation on the collected radiation would be exponential in the black hole entropy i.e., $10^{10^{80}}$ more years to perform this calculation.
- Can we create smaller "microscopic" Black Holes in the lab to avoid this?
- Sure…if we can push constituents within their mutual Schwarzschild Radius. For $O(1)$ constituent this requires $E_p \sim 10^{19}$ GeV which exceeds energy scales of the Large Hadron Collider by 10^{15} magnitude.

Best test objects?

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Are Black Holes like Vegas? What happens in a Black Hole, stays in a Black Hole?

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Black Holes Exist (we literally have a picture). The world is fundamentally quantum. Hence, nature must find a way to accommodate both.

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How do we do this? Thought experiments? Indirect Evidence? Math? Comp Sci?

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How do we do this? Thought experiments? Indirect Evidence? Math? Comp Sci?

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“Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum, and by golly it's a wonderful problem, because it doesn't look so easy.” - R. P. Feynman

AMPS Firewall

BH: Unitary Quantum Systems

- Almheiri, Marlot, Polchinski, Sully (AMPS 2012): Observer's experience while entering a black hole.
- QFT implies short-ranged entanglement in the vacuum.
- Bell-Pairs straddling around the event Horizon.
- If the observer doesn't see this then they see a wall of Planck-energy photons that will disintegrate them.

BH: Unitary Quantum Systems {Information Theoretic View}

Assumption: Hawking radiation “should” carry information about the infalling matter.

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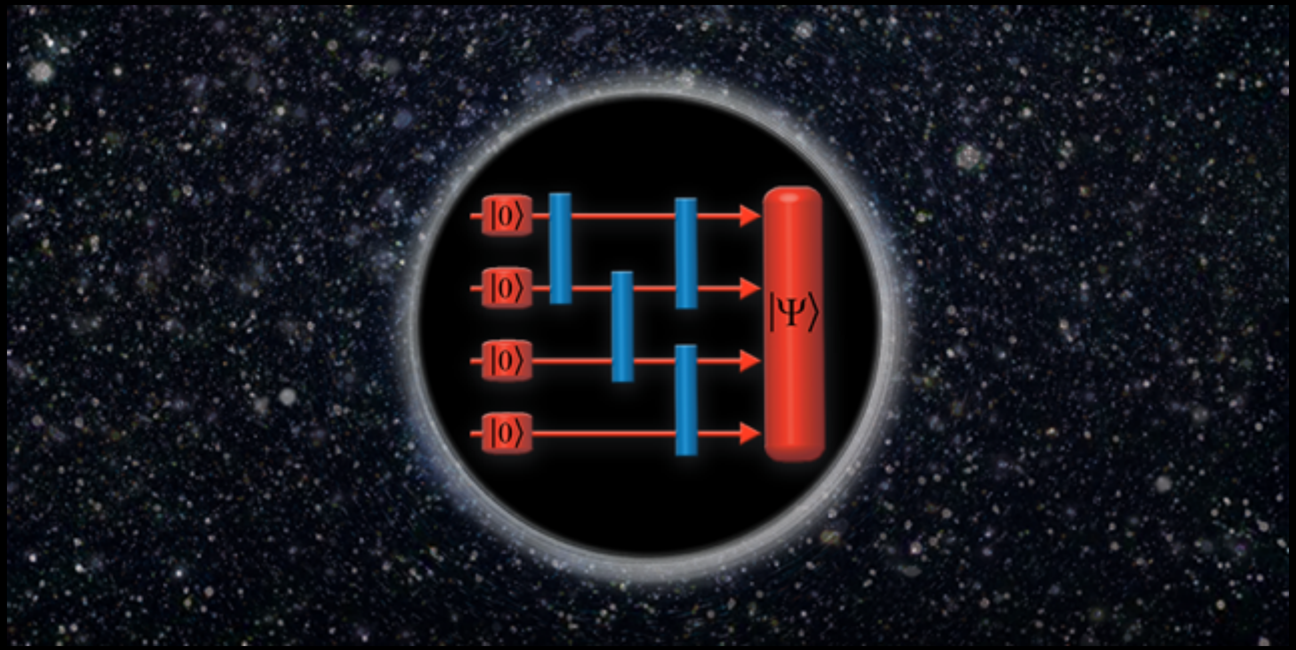
Block Hole Evolutionary Dynamics is Unitary (Viewed from Outside or even created by hand)

$$UU \ldots U \,|\, \psi\rangle$$



$$tr(\rho_{\text{initial}})^2 = 1$$

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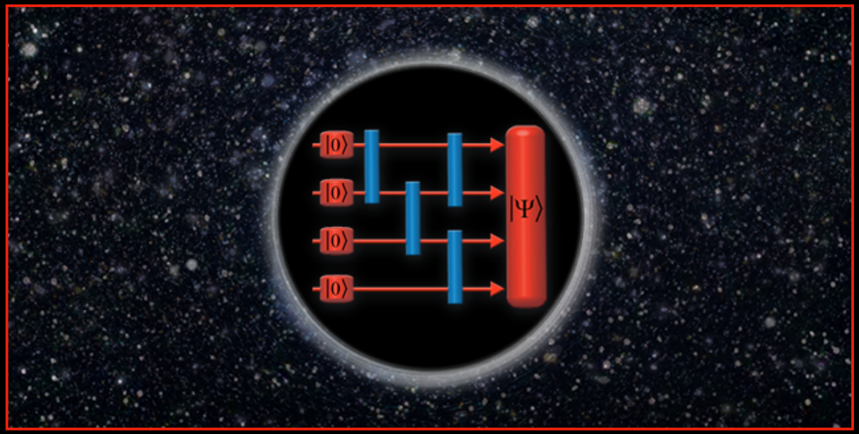
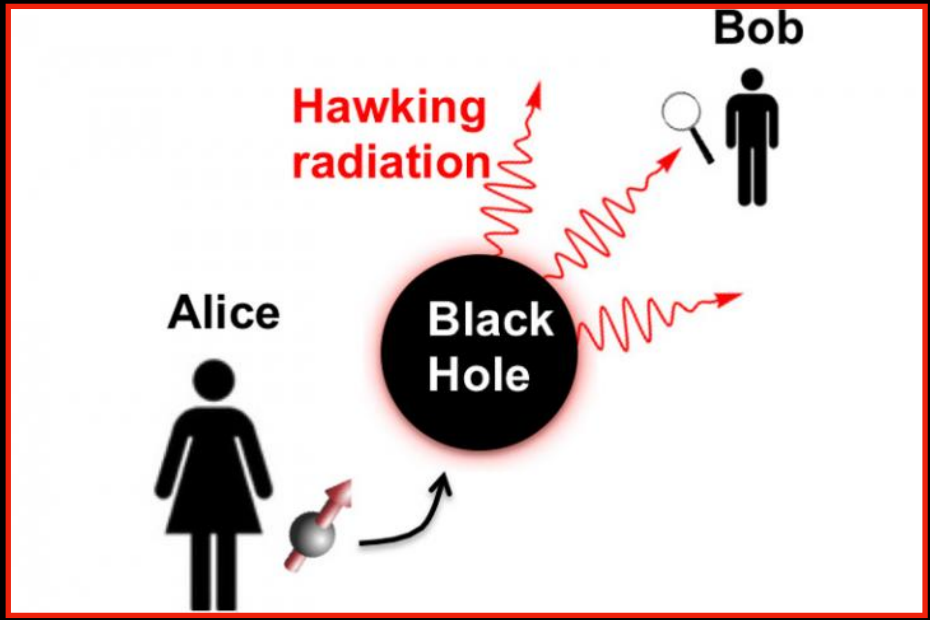




$$tr(\rho_{\text{initial}})^2 = 1 \xrightarrow{\text{Evolution}} tr(\rho_{\text{rad}})^2 < 1$$

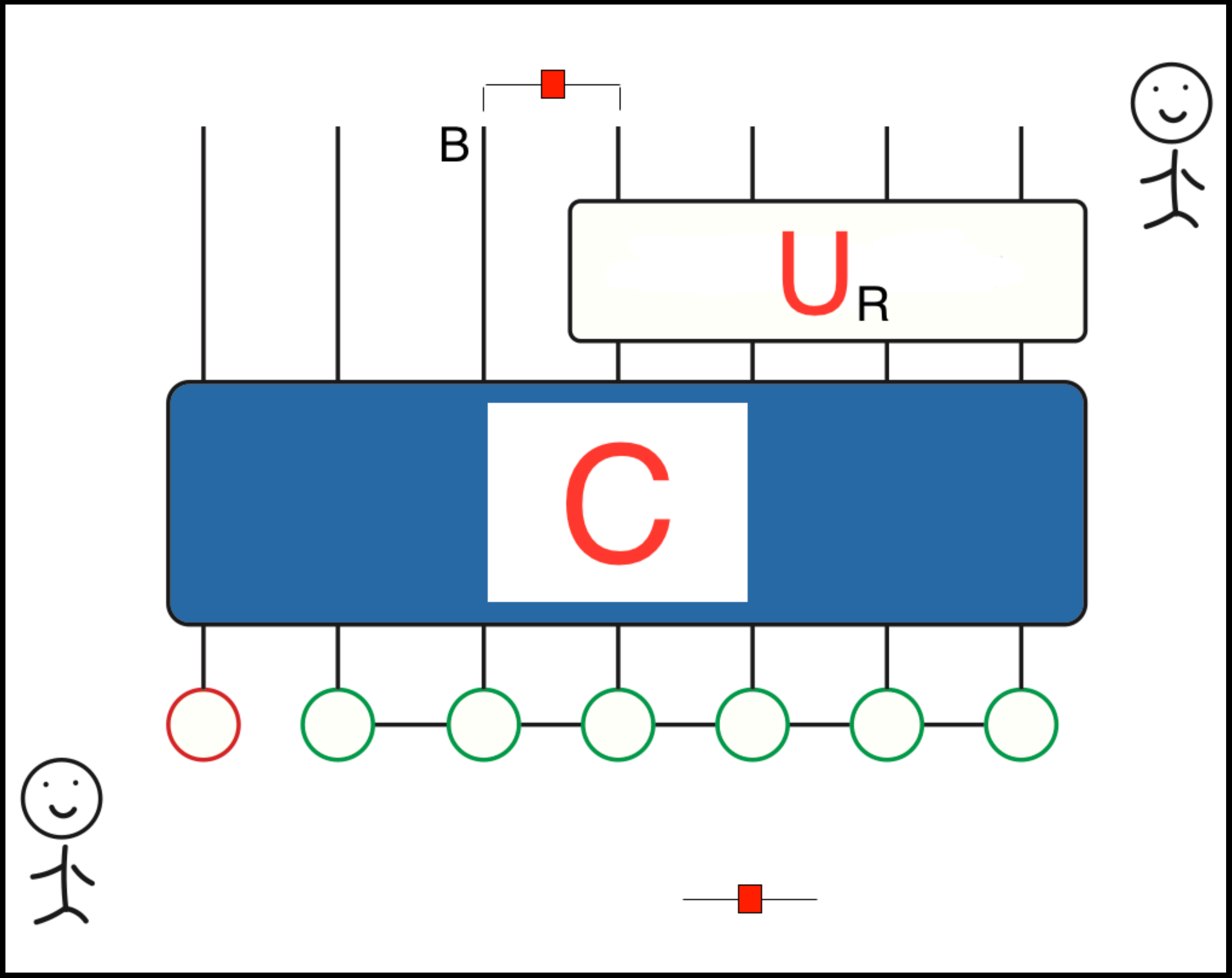
Mind=Blown!!

BH: Unitary Quantum Systems {Information Theoretic View}

- Let "a Black Hole" be described by $|00\ldots0\rangle$ (n qubits in this state)

- Since Black Holes are fast scramblers, we want its formation to from a Random quantum circuit and this is also valid because we don't have access to any interior degrees of freedom or dynamics.

- Bob outside has access to the ρ_{rad} we defined before.



BH: Unitary Quantum Systems {Information Theoretic View}

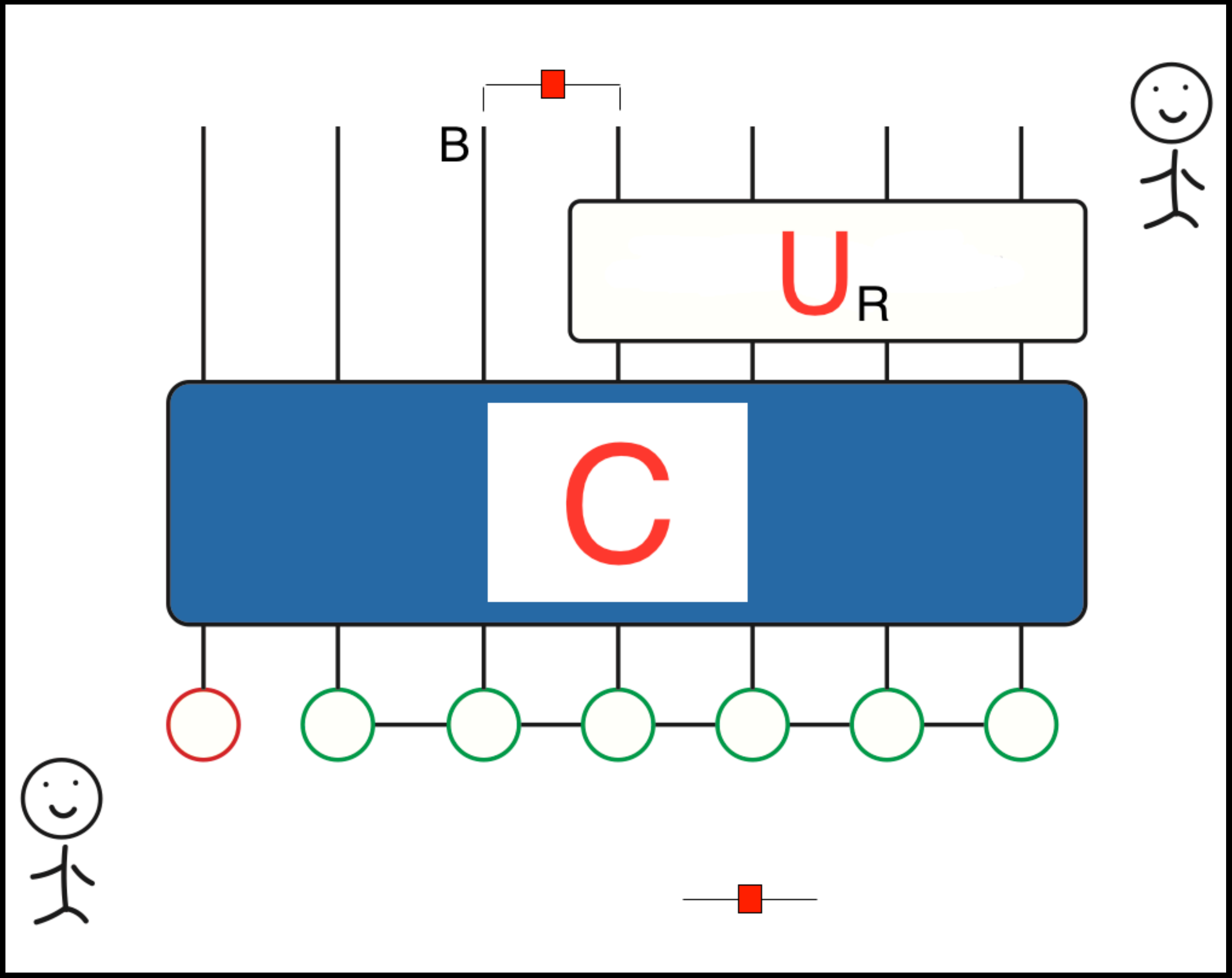
- Bob's accessibility is physically restricted the radiation coming out. If he has access to the first k -qubits (as they come out).

- The two regions of interest are

  1. $k < n/2$

  2. $k > n/2$

- Calculating the reduced density matrix ρ_{rad} (by tracing out the qubits still inside) we see that for the first case $rank(\rho) = 2^k$ and for the second case $rank(\rho) = 2^{n-k} < 2^k$.

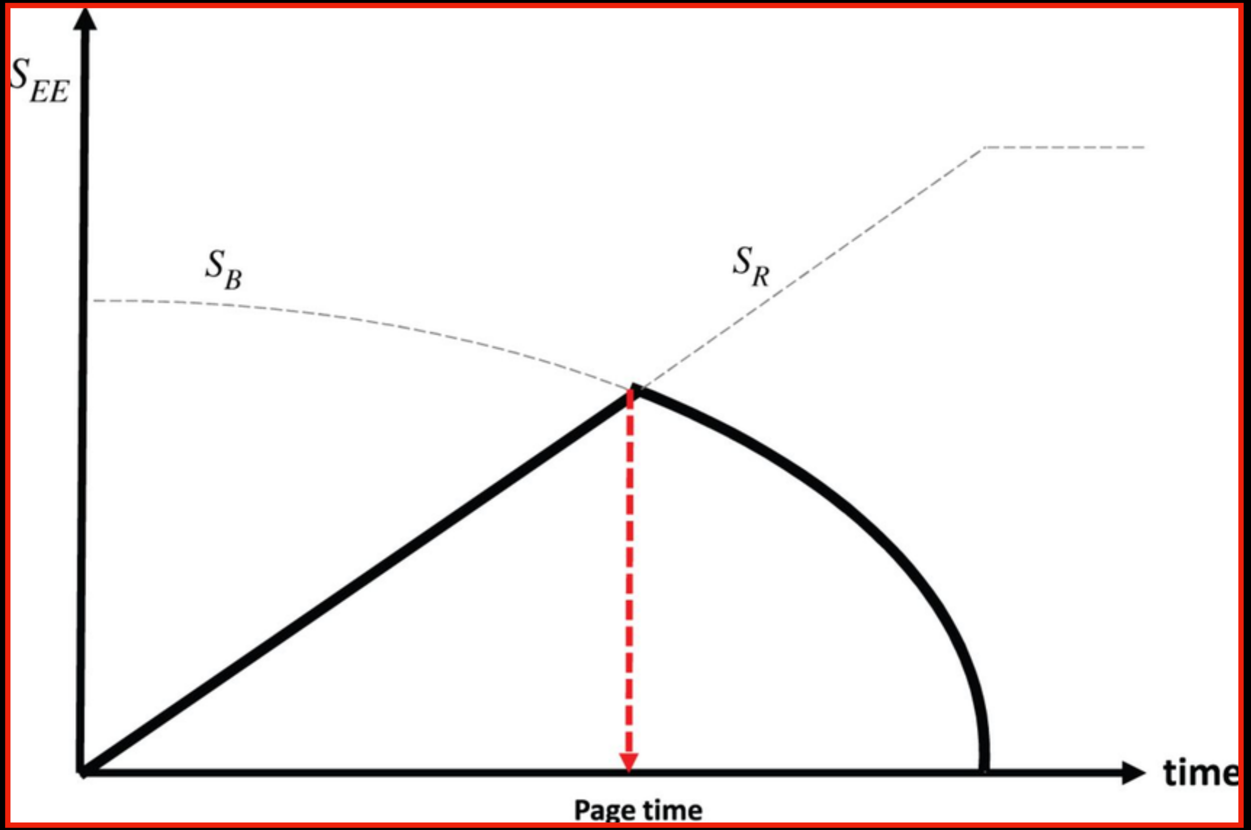


$$\rho_{\text{rad}} = \sum_i^{2^{n-k}} p_i |\psi_i\rangle\langle\psi_i|$$

*$rank(\rho) = 1$ for a pure state

BH: Unitary Quantum Systems {Information Theoretic View}

- The reduced density matrix is no longer maximally mixed.
- This indicates that when exactly half of the qubits emerge out of the black hole, the outside observer let's say Alice, can access the correlations between the Hawking photons and the infalling matter. This is conjectured to happen after “Page Time”.

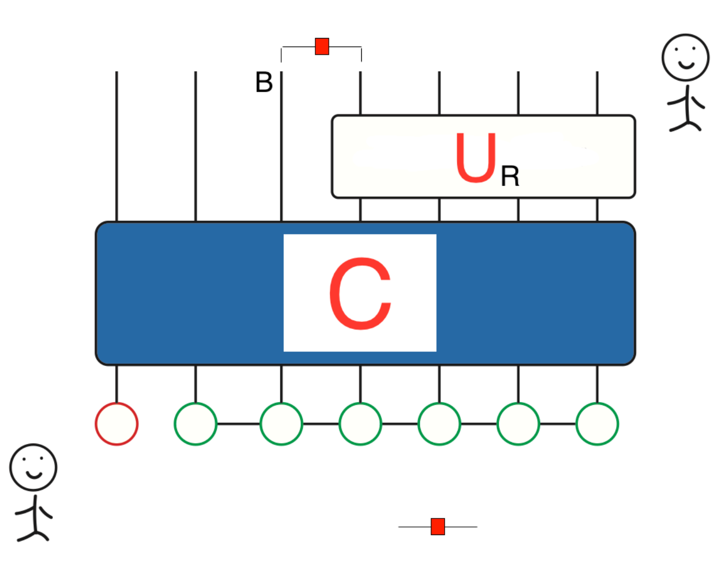


Notice the sharp transition from

$$k < n/2 \longrightarrow k > n/2$$

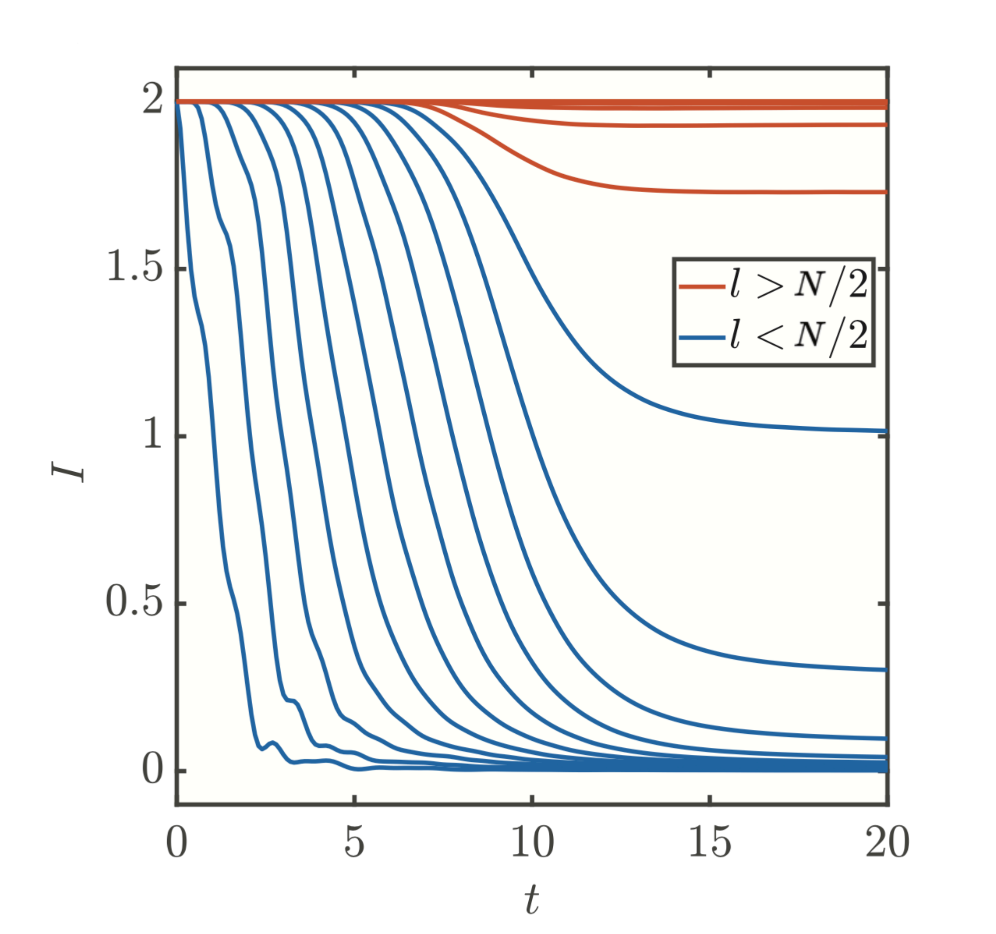
BH: Unitary Quantum Systems {Information Theoretic View}

- We can reinforce this striking transition. By doing a calculation on a system of n spins, and keeping track of entanglement propagation using mutual information:



$$I(B:R) = S(B) + S(R) - S(BR)$$

$$0 \leq I(B:R) \leq 2\,min(S(B), S(R))$$



- The long-time behaviour lets Bob access this scrambled information.

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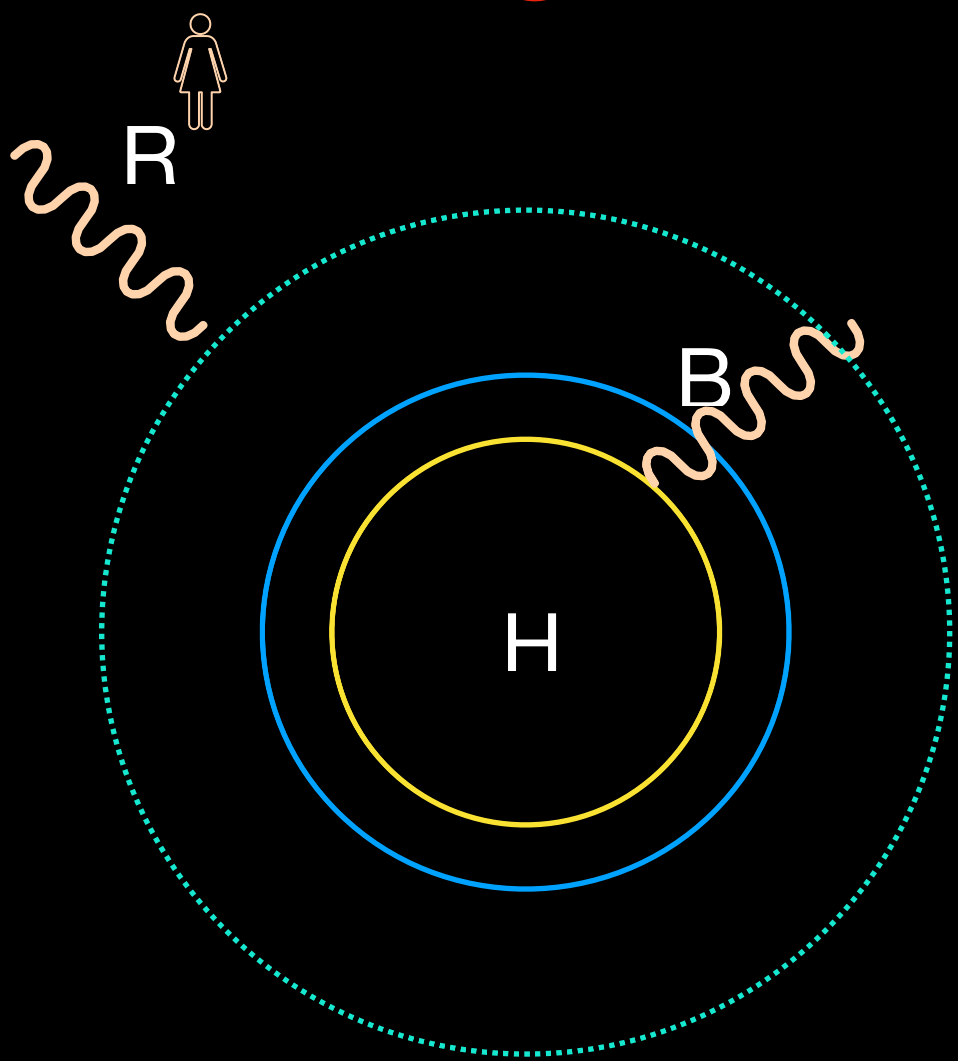
D. Harlow, P. Hayden(2013): Black Holes and Computational Complexity are related.

Harlow-Hayden Arguments

Revisiting the Paradox



R: Far away radiation.

H: Interior of the the Black Hole.

B: Radiation (1 qubit) just coming out.

Harlow-Hayden Arguments

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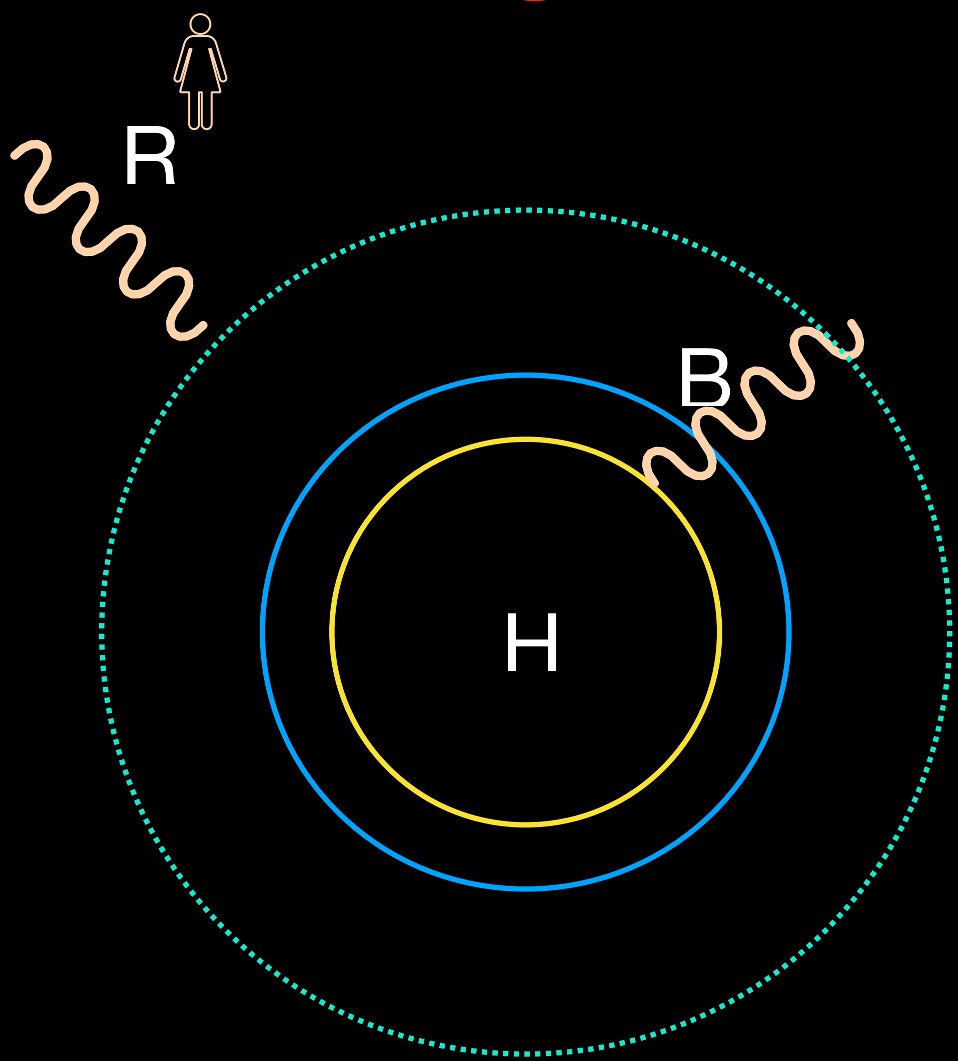

- Expect that B is entangled with R.
- ρ_{rad} is not maximally mixed.
- Alice puts the leftmost qubit of R in a Bell pair with B (maximally entangled).

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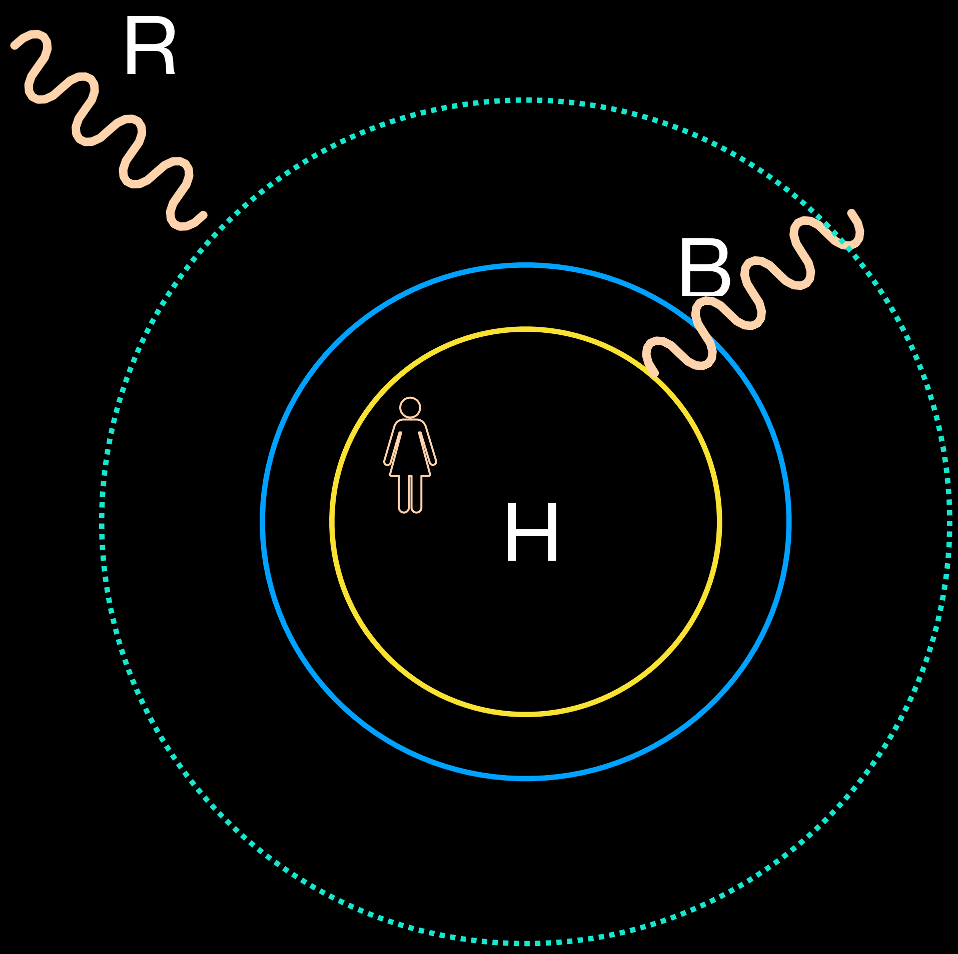

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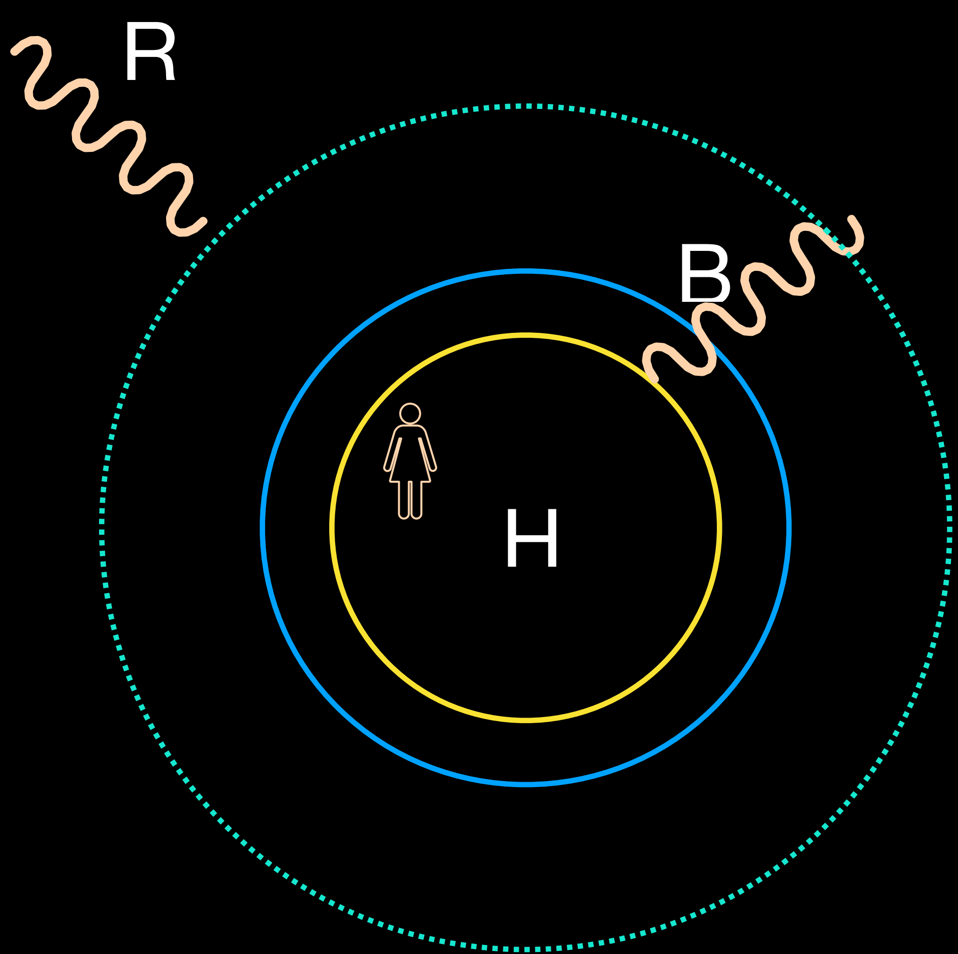
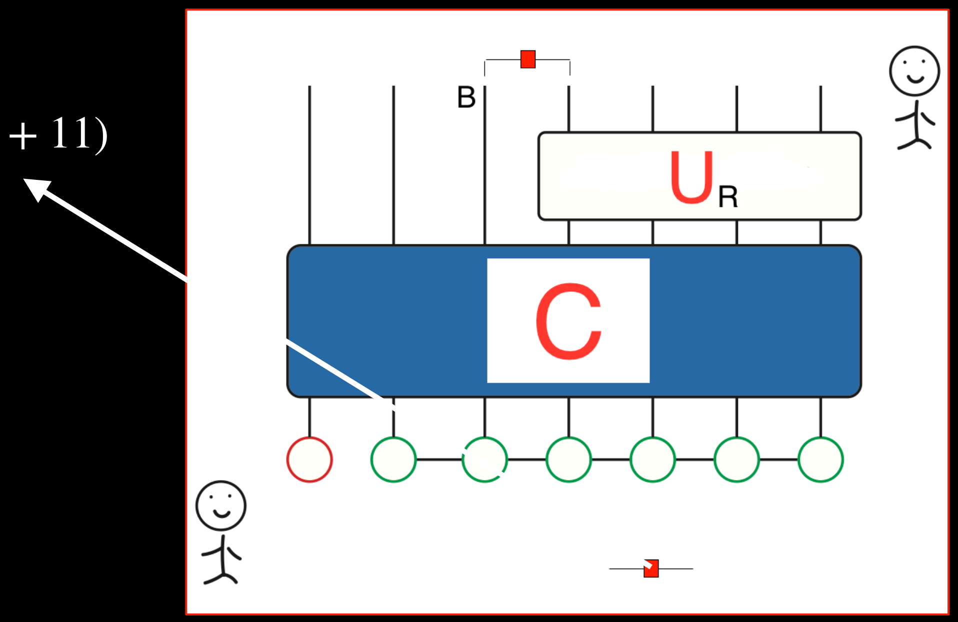


- Recall qubit B is maximally entangled with H (AMPS)
- Alice should observe this entanglement as well.
- Violation of Monogamy of Entanglement

$$\frac{1}{\sqrt{2}}(00 + 11)$$



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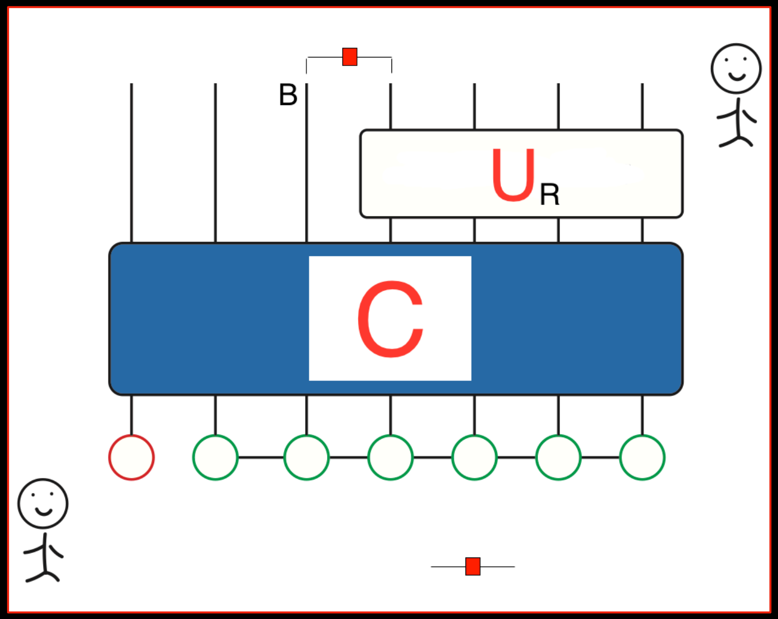
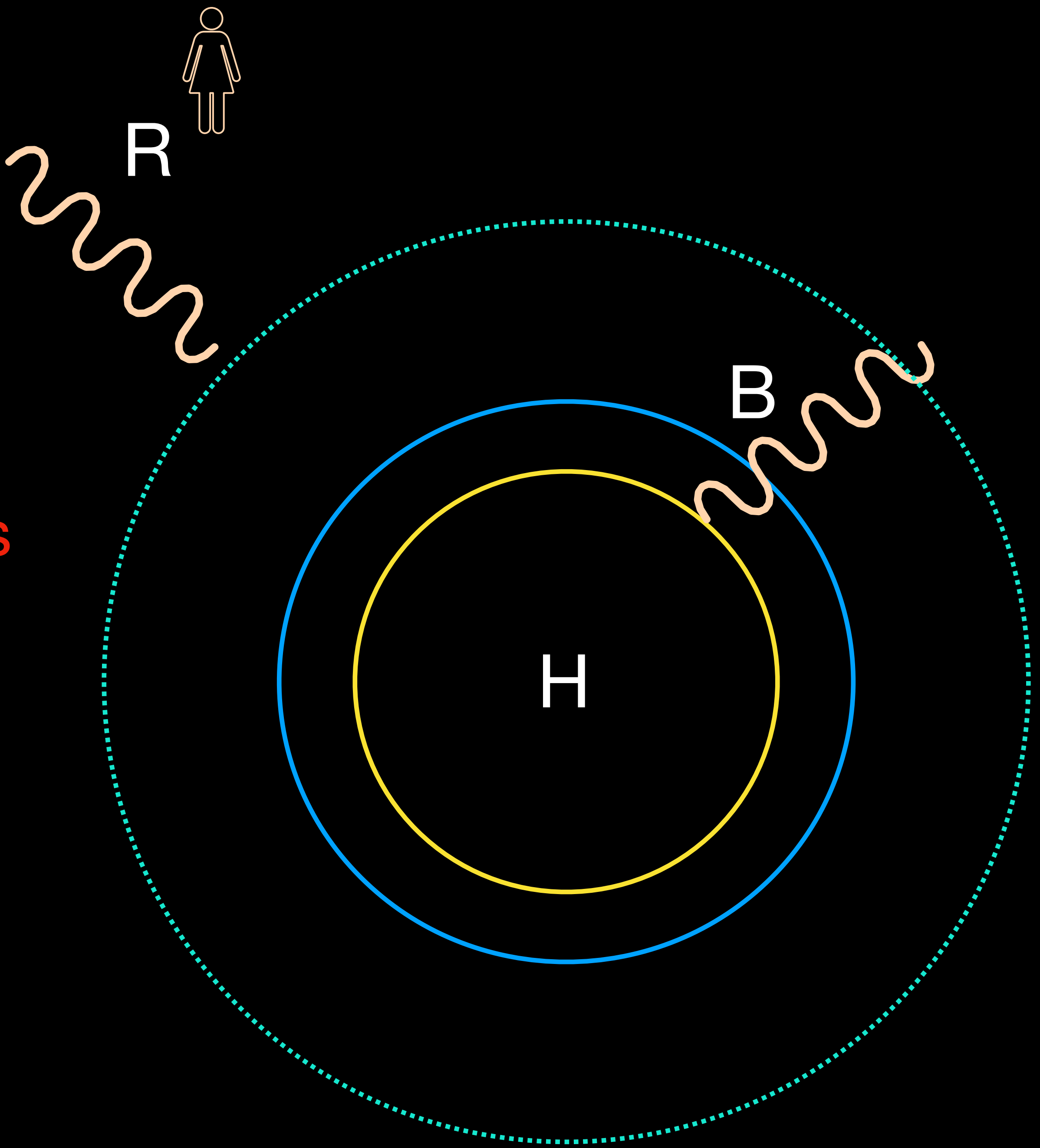
Harlow-Hayden Arguments

Decoding Task

Consider now the output state of this circuit C to be a tri-partite state $|\psi\rangle_{RBH}$. The task is to use a unitary U to entangle R and B. This is doable because we're already guaranteed this unitary.





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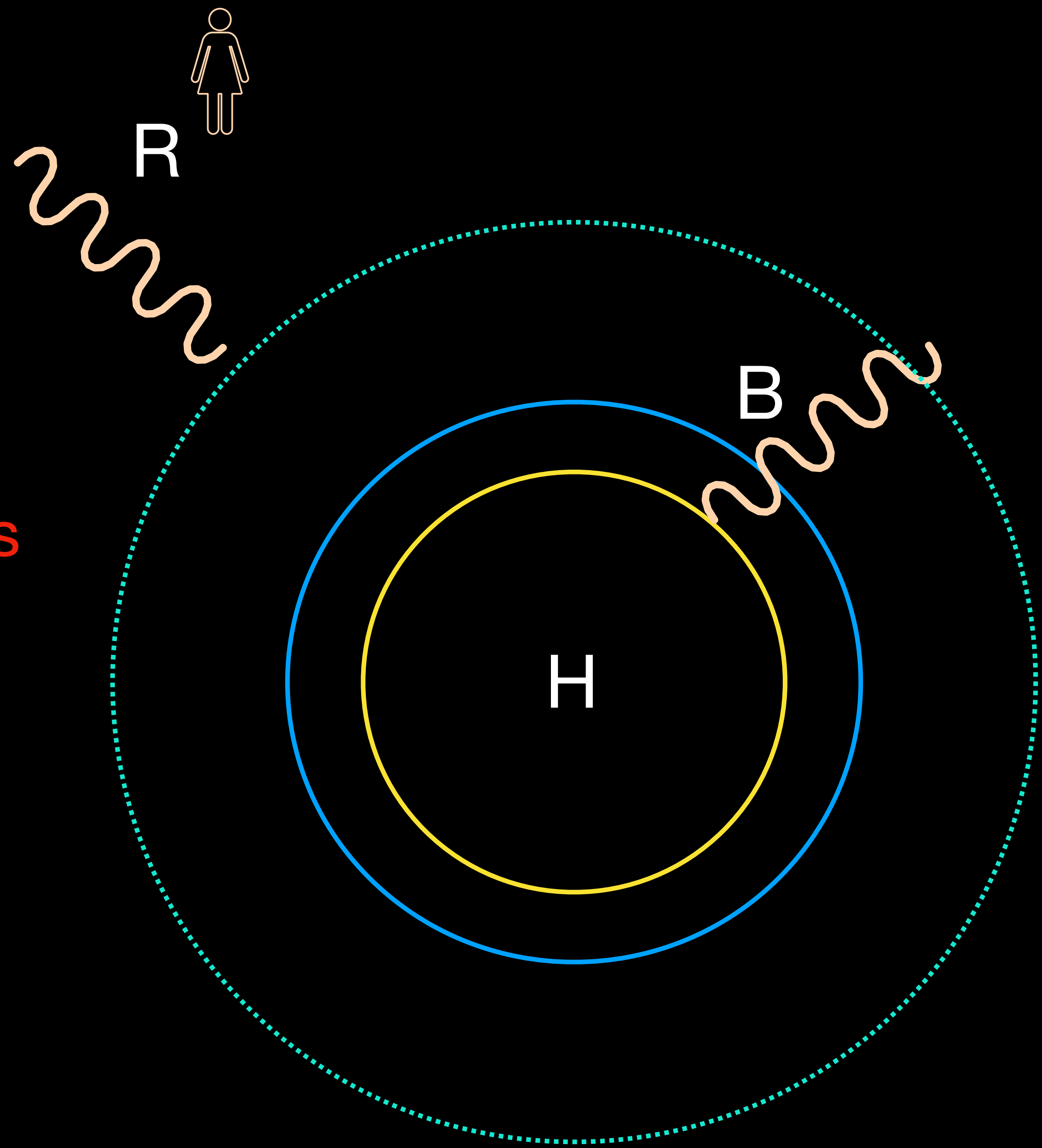

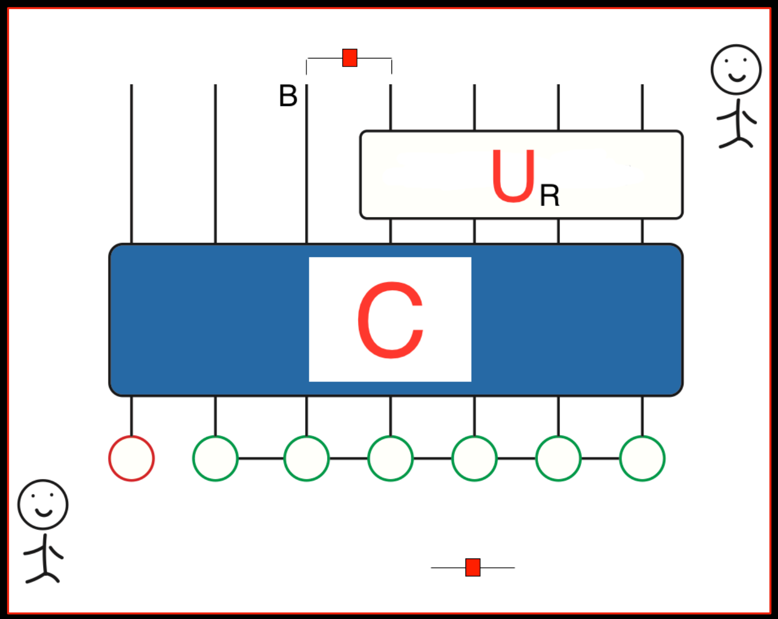

Is this even computationally tractable?

Harlow-Hayden Arguments
Decoding Task

Recall we want

- R has $k > n/2$
- ρ_{RB} is not maximally mixed
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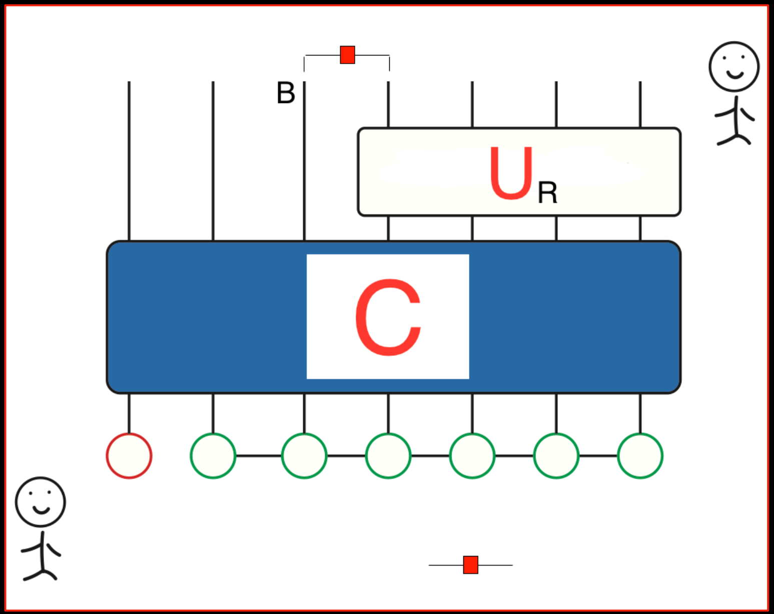
- If we use a quantum circuit to do this, this circuit will have an exponential size in terms of number of gates and time of computation.



Harlow-Hayden Arguments

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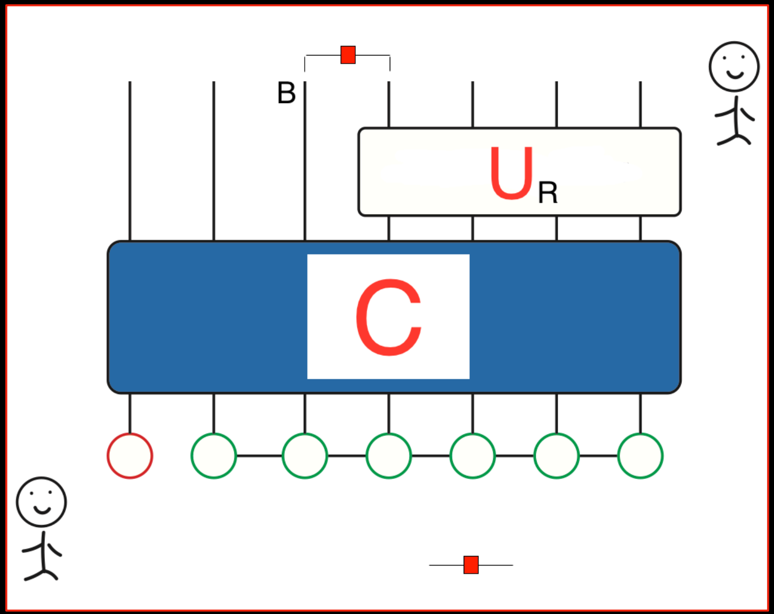


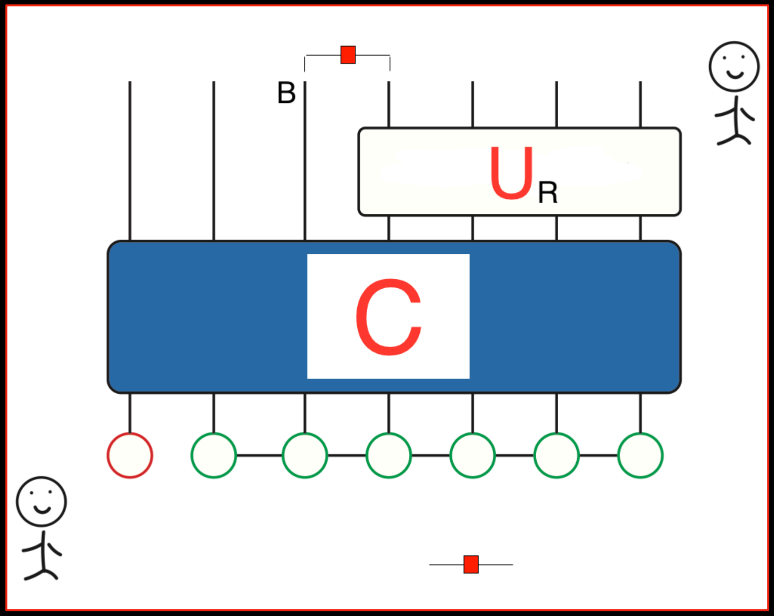
Easy:

Just invert C!

Harlow-Hayden Arguments

Decoding Task

- Physically means we wait for the Black Hole to evaporate. That means no paradox at all.
- When the Black Hole is $\sim 50\,\%$ evaporated, there exists a U_R that can distill the entanglement between R and B (Page's argument).
- However the task is to find out how to efficiently do this.



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Harlow-Hayden Arguments
Decoding Task

↓

Theorem: If the HH task can be done in polynomial time $p(n)$, then SZK ⊆ BQP

Harlow-Hayden Arguments

Complexity Theory in A Minute

- Measure of Complexity: amount of resources (space and time) required by an Algorithm.

- Polynomial (P): $t \sim O(N^k)$

- Non-Degenerate Polynomial (NP): Given a solution, it can be verified in polynomial time.

- Bounded Error Quantum Polynomial Time(BQP): $\{C_n\}_{n \geq 1}$ acting on $p(n)$ qubits over some finite gate set. Problems that can be solved in polynomial time using a quantum computer. The Toffoli gate tells us that all classical calculation can be done on a quantum compute this mean $P \subseteq BQP$ with bounded error of at most 1/3.

- Statistical Zero Knowledge (SZK): Class of decision problems for which a “yes” answer can be verified by a statistical zero-knowledge proof protocol. By exchanging messages with the prover, the verifier must become convinced (with high probability) that the answer is indeed “yes” without learning anything about the problem statistically.

Harlow-Hayden Arguments
Decoding Task

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Theorem: If the HH task can be done in polynomial time $p(n)$, then SZK $\subseteq$ BQP

↓

If HH task can be done in $t \sim p(n)$ then a problem called set equality can also be solved in quantum polynomial time.

Harlow-Hayden Arguments

Set Equality

Given a Black Box Access to two injective maps (not permutational symmetric) $f, g : \{0,1\}^n \to \{0,1\}^{p(n)}$

- $Range(f) = Range(g)$, or
- $Range(f) \cap Range(g) = \emptyset$

Harlow-Hayden Arguments

Set Equality

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- $Range(f) = Range(g)$, or
- $Range(f) \cap Range(g) = \emptyset$

The problem is to decide which?

Harlow-Hayden Arguments

Set Equality → HH Task

Theorem: Any Quantum Algorithm (in this Black Box setting) for Set Equality must make $\Omega^{1/3}$ queries. (Zhandry et al.)

- Let $p(n)$-size circuit C prepare the following state (provided it can compute f and g)

$$|\psi\rangle_{\text{RBH}} = \frac{1}{2^{n-1}} \sum_{x\in\{0,1\}^n} \left(|x,0\rangle_R |0\rangle_B |f(x)\rangle_H + |x,1\rangle_R |1\rangle_B |g(x)\rangle_H \right)$$

Harlow-Hayden Arguments

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Case 1: $Range(f) \cap Range(g) = \varnothing$. H register decoheres any entanglement between R and B, like if H had measured B. Implies ρ_{RB} is not entangled. Thus, HH task is violated.

Harlow-Hayden Arguments

Set Equality → HH Task

Case 2: $Range(f) = Range(g)$. Alice acts on R with following maps.

$$Id : |x,0> \to |x,0>$$

$$|x,1> \to |f^{-1}(g(x)),1>$$

$$\implies |\psi>_{\text{RBH}} = \frac{1}{2^{n-1}} \sum_{x\in\{0,1\}^n} \left(|x,0>_R |0>_B + |x,1>_R |1>_B \right) |f(x)>_H$$

Harlow-Hayden Arguments

Set Equality → HH Task

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Therefore, as a recap if the HH task was easy, given f and g for which we wanted to solve set-equality, we can start by preparing the above state, apply the unitary (≡ doing HH), then finally projecting onto the Bell state to check if we succeeded. For Case 2, as we can see we would succeed with probability 1. For Case 1, we would succeed with probability at most 1/2. Thus, we can decide with bounded error probability, whether we want to choose Case 1 or Case 2. If set equality is hard for a quantum computer then so is the HH decoding task.

Final Remarks

- The role of Black Holes in all the above arguments is to scramble our prepared quantum system and provide an inaccessible region of spacetime. This restricts our quantum computation to any region outside the black hole. Typically other chaotic physical systems do not have the property of violating the principle of monogamy of entanglement.

- The HH task tells us that the current understanding of “effective” or approximate theories, namely Quantum Field Theory and Gravity, work well under certain circumstances fail as soon as we're able to solve a problem in polynomial time which is currently considered exponentially hard. This hints at a more complete quantum theory of gravity is currently missing from the framework. Previously it was known that these theories fail at very high energies (UV regimes) and large curvatures (Big Bang and Black Holes), however the HH argument gives us another regime of failure which is the regime of exponential computational complexity.

Coming Full Circle: AdS/CFT

- Recall I started this report by talking about the importance of Dualities in modern physics.

- The most prominent example is the AdS/CFT duality.

- equivalence between a $D+1$ dimensional quantum theory of gravity living in the bulk of asymptotically AdS space and a “regular” quantum theory in D spacetime dimensions living on the boundary of this AdS space.